Roll No. _____ Code No. Q-4394

FOURTH SEMESTER EXAMINATION 2021-22

M.Sc. MATHEMATICS

Paper - II

Partial Differential Equations, Mechanics and Gravitation-II

Time : 3.00 Hrs. Max. Marks : 80

Total No. of Printed Page : 03 Mini. Marks : 29

Note:- Question paper is divided into three sections. Attempt question of all three section as per direction Distribution of marks is given in each section.

Section 'A'

Very short answer question (in few words)

Q.1 Attempt any six questions from the following : 6x2=12

(i) Define constraints.

(ii) Define Generalised momentum.

(iii) Define Routhean function.

(iv) Define Hamiltonian.

(v) What is canonical transformation ? Explain.

(vi) What in surface density ? Explain.

(vii) Write the formula for potential of their uniform infinite rod.

(viii) Define Equipotential surface.

(ix) Write the formula for potential of spherical shell of finite thickness.

(x) Define Lagranges Bracket.

Section 'B'

Short answer type question (in 200 words)

Q.1 Attempt any four questions from the following : 4x5=20

(i) Find the shortest distance between two points in a plane.

(ii) Show that the generalised momentum conjugate to a cyclic co-ordinate is conserved.

(iii) Derive the relation between attraction and potential.

(iv) Find a curve joining two points along which a particle falling from rest under the influence of gravity travels from higher to the lower point in the minimum time.

(v) Explain non-Halonomic systems.

(vi) By using Hamiltons canonical equation

Show that the tranformation

$P = \frac{1}{2}\left(p^2 + q^2\right)$ and $Q = \tan^{-1}\frac{q}{p}$ is canonical.

(vii) Find the potential of solid sphere at an external point.

Section 'C'

Long answer/Essay type question. **4x12=48**

Q.3 Attempt any four questions from the following questions :

(i) Derive Hamilton canonical Equation.

(ii) If $[\phi,\psi]$ in the poisson Bracket of ϕ & ψ then prove that :

(a) $\frac{\partial}{\partial t}[\phi,\psi] = \left[\frac{\partial\phi}{\partial t},\psi\right] + \left[\phi,\frac{\partial\psi}{\partial t}\right]$

(b) $\frac{d}{dt}[\phi,\psi] = \left[\frac{d\phi}{dt},\psi\right] + \left[\phi,\frac{d\psi}{dt}\right]$

(iii) (a) Show that Lagrange's Bracket is invariant under a canonical transformation.

(b) Prove that Lagrange's Bracket does not obey the commutative law.

(iv) State and prove Jacobi-Poission theorem.

(v) Find the potential of their spherical shell at an external and internal points.

(vi) Show that the potential of a uniform circular disc, of man M and radius a, at a point in its plane distant c from its centre is :

$$\frac{4\gamma M}{\pi a^2}\int_0^{\frac{\pi}{2}} \sqrt{a^2 - c^2 \sin^2\theta}\ d\theta$$

or

$$\frac{4\gamma M}{\pi a^2}\int_0^{\sin^{-1}\frac{a}{c}} \sqrt{a^2 - c^2 \sin^2\theta}\ d\theta$$

according as c is less or greater than a.

(vii) Find the potential of circular disc on unit man placed at a point on the axis, perpendicular to the plane of the disc which panes through its centre.

--00--